A method for designing impulsive low-energy transfers between the Earth and the Moon

Stephen DeSalvo[1], Jonathan Essen, Ken Ho, Gwenan Knight

IPAM-RIPS 2006: Jet Propulsion Laboratory team

August 18, 2006

[1] Project manager

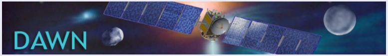
Outline

1. Introduction
   - Project sponsor
   - Low-energy Earth-Moon mission design
2. Approach
   - Approximation as two coupled three-body systems
   - Four-body mission design using three-body manifolds
3. Method
   - Invariant manifold computation
   - Transfers between Sun-Earth and Earth-Moon manifolds
   - Mission design using invariant manifolds
4. Results
5. Conclusion

Introduction
Project sponsor: Jet Propulsion Laboratory

- A NASA center, staffed and managed for the government by Caltech

Sample missions



Images from JPL: `http://www.jpl.nasa.gov/`.

Introduction

Project sponsor: Martin Lo

- Industry liaison: Martin Lo
  - JPL mission analyst
  - Designer of the Genesis Discovery Mission
  - Low-energy mission design using dynamical systems theory

Genesis Discovery Mission

Unique trajectory requiring only very small mid-course corrections!

Image from JPL: http://www.jpl.nasa.gov/.

Introduction

Low-energy transfers

- Simple dynamics: the Hohmann transfer

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- The Δv cost function

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Why do we want a low Δv?

A lower Δv means a lower fuel requirement—a lower fuel requirement means more space for scientific equipment and other cargo!

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What is an invariant manifold?

An invariant manifold is a "gravitational passageway" that connects possibly very distant regions of space and guides the behavior of nearby trajectories.

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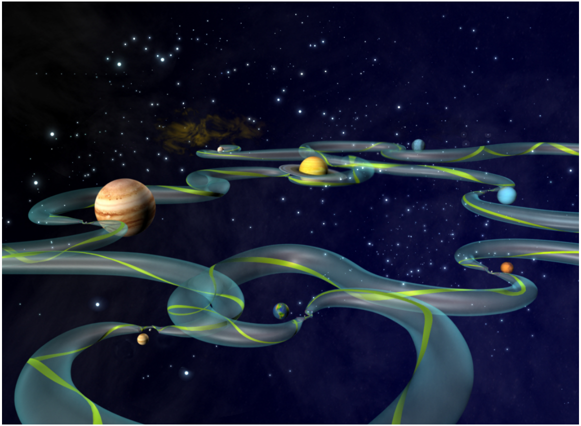
- The interplanetary superhighway

Introduction
The interplanetary superhighway



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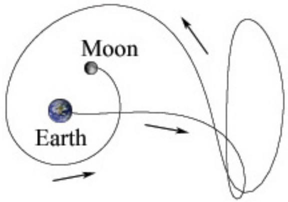
Introduction

Objective: a method for low-energy Earth-Moon mission design

- Low-energy Earth-Moon mission design using invariant manifolds



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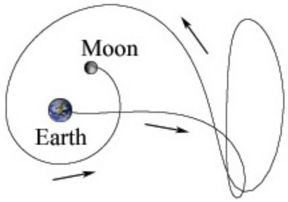
Objective: a method for low-energy Earth-Moon mission design

- Low-energy Earth-Moon mission design using invariant manifolds
    - No single manifold joining the Earth and the Moon
    - Transfer by traveling on connecting manifolds



Introduction

Objective: a method for low-energy Earth-Moon mission design

- Low-energy Earth-Moon mission design using invariant manifolds
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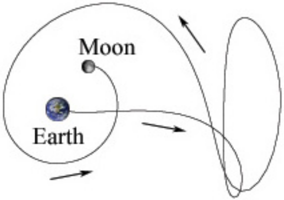
    - Transfer by traveling on connecting manifolds
- Study of manifold transfers



Introduction
Project deliverables

- MATLAB toolbox
  - Compute invariant manifolds
  - Investigate manifold intersections
  - Construct end-to-end trajectories

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- Sample trajectories designed with the method

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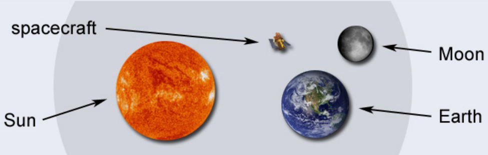
- Manifold database and visualization routines
- Study of manifold transfers
- Sample trajectories designed with the method
- Recommendations for future research

Approach

Approximation as two coupled three body-systems

The four-body problem



Approach

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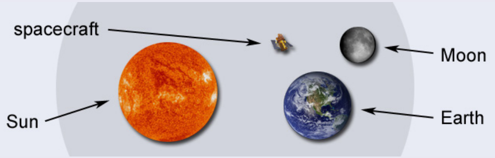
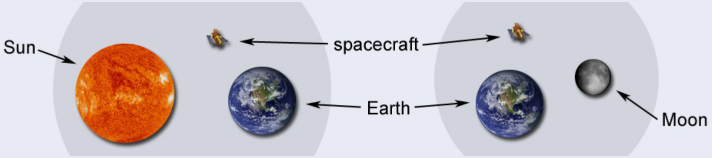
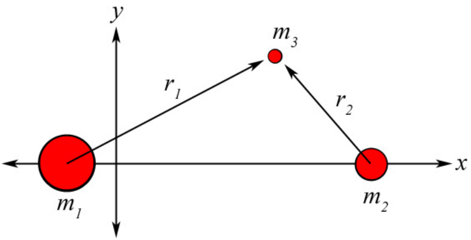


Approximation as two coupled three-body problems



Approach
The planar circular restricted three-body problem

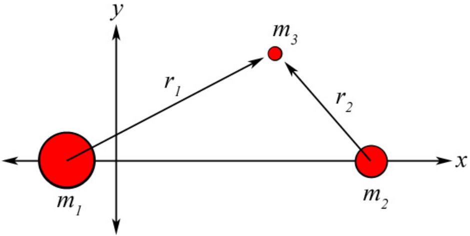
- PCR3BP assumptions



Approach

The planar circular restricted three-body problem

- PCR3BP assumptions
- The rotating frame



Approach

The planar circular restricted three-body problem

- PCR3BP assumptions
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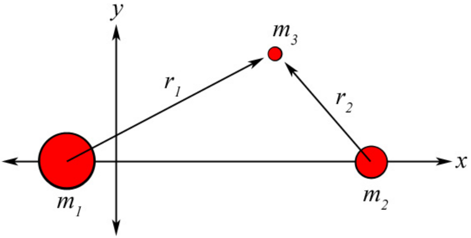
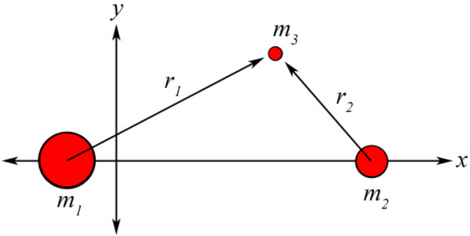
- Parameterization: the Jacobi constant



Approach

The planar circular restricted three-body problem

- PCR3BP assumptions
- The rotating frame
- Parameterization: the Jacobi constant
- Two PCR3BPs: Sun-Earth-SC and Earth-Moon-SC



Approach

Four-body mission design using three-body manifolds

- Use invariant manifolds generated by three-body dynamics

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Four-body mission design using three-body manifolds

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- Extensive previous studies of the manifolds of the PCR3BP (Poincaré, Conley, McGehee)
- Connect systems by manifold transfer
- Trajectories following three-body manifolds approximate trajectories in the full system
- Correct in four-body dynamics

Method

Invariant manifold computation

- Method overview
  1. Computing invariant manifolds
     - Lagrange points
     - Periodic orbits
     - Invariant manifolds
  2. Better manifolds: use continuation methods to find larger periodic orbits
  3. Getting onto a manifold: transfer to/from circular orbit
  4. Transferring between manifolds
  5. Putting it all together: mission design

Method

Invariant manifold computation: Lagrange points

- Equilibrium points

Method

Invariant manifold computation: periodic orbits

- Linearize to approximate periodic orbits and correct

Method

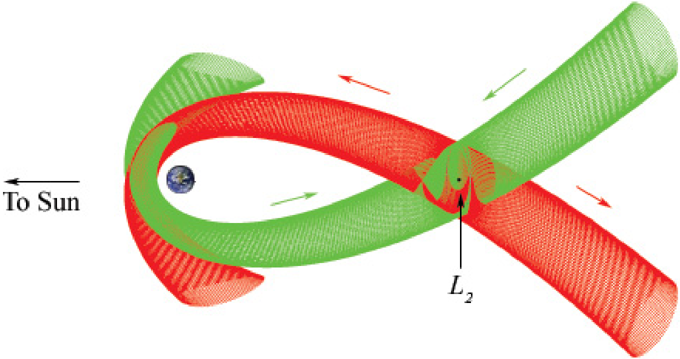
Invariant manifold computation: invariant manifolds

- Perturb in stable/unstable directions and integrate

Method

Invariant manifold computation: invariant manifolds

- Perturb in stable/unstable directions and integrate



Method
Continuation methods

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Method

Continuation methods for periodic orbits I

- Linearization plus correction breaks down for large orbits

Method

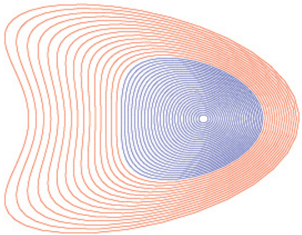
Continuation methods for periodic orbits I

- Linearization plus correction breaks down for large orbits
- Continuation methods

Method

Continuation methods for periodic orbits I

- Linearization plus correction breaks down for large orbits
- Continuation methods
  - Periodic orbits vary continuously
  - Extrapolate from initial conditions obtained by linearization
  - Correct extrapolations



Method

Continuation methods for periodic orbits II

Earth-Moon-SC

- Linearization: $A_x = 0.04$
- Continuation: $A_x = 0.0742$
- Increase: $\Delta A_x = 0.0342$
- Percent increase: 86%

Sun-Earth-SC

- Linearization: $A_x = 0.00065$
- Continuation: $A_x = 0.001865$
- Increase: $\Delta A_x = 0.001215$
- Percent increase: 187%

Significantly more powerful way of generating periodic orbits!

Method

Transfers between circular orbits and manifolds

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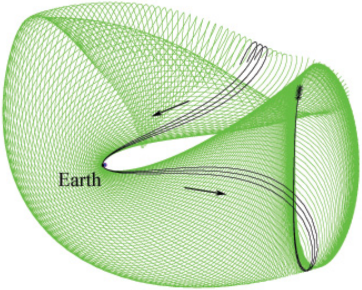
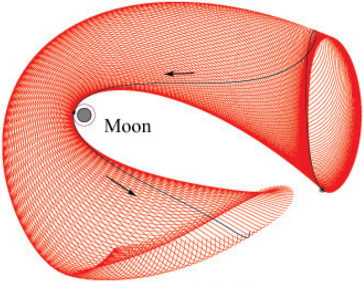


Sun-Earth rotating frame



Earth-Moon rotating frame

Method

Transfers between Sun-Earth and Earth-Moon manifolds

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Method

Transfers between Sun-Earth and Earth-Moon manifolds: Poincaré sections

Poincaré sections

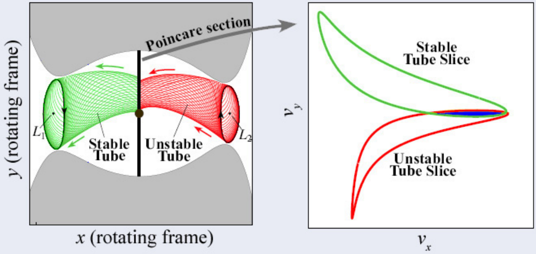


Image adapted from Ross (2002)

Method

Transfers between Sun-Earth and Earth-Moon manifolds: Poincaré sections

Poincaré sections

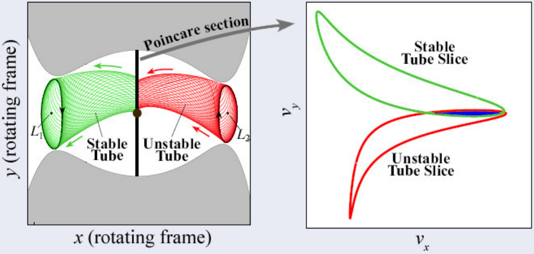


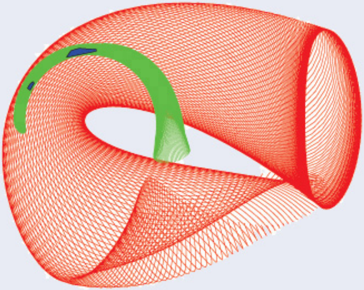
Image adapted from Ross (2002)

- Advantage: simple visualization of manifold intersections
- Disadvantage: arbitrariness of hyperplane cut

Method

Transfers between Sun-Earth and Earth-Moon manifolds: transfer region boundaries I

Our approach: transfer region boundaries



- Find regions of low-energy transfer
- Spatial intersections of trajectories
- Project into velocity space
- Types of transfer regions
- More complete view of Δv

Method

Transfers between Sun-Earth and Earth-Moon manifolds: transfer region boundaries II

- Automated transfer analysis

Method

Transfers between Sun-Earth and Earth-Moon manifolds: transfer region boundaries II

- Automated transfer analysis
- Database analysis

Method

Transfers between Sun-Earth and Earth-Moon manifolds: transfer region boundaries II

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  - Metrics: areas of regions
  - Ranking of manifold pairs

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Transfers between Sun-Earth and Earth-Moon manifolds: transfer region boundaries II

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Example

1. Find manifold pair with large transfer region
2. Intersection of minimal Δv: guess for transfer point

Method

Mission design using invariant manifolds

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Mission design using invariant manifolds

- Use manifold transfers to design a trajectory piecewise

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Mission from low Earth orbit to lunar orbit

1. Low Earth orbit to Sun-Earth periodic orbit
2. Sun-Earth periodic orbit to Earth-Moon periodic orbit
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- Trajectory pieces: initial guess for a mission
- More realistic mission: correct in four-body system
  - Sample points from patched trajectory
  - Formulation as an optimization problem

Results
MATLAB toolbox

Manifold computation

- PCR3BP equations
- Lagrange points
- Periodic orbits
  - Linearization
  - Continuation methods
  - Differential correction
- Stability directions
- Manifold propagation
  - High-order integration
  - Adaptive perturbation

Manifold transfer

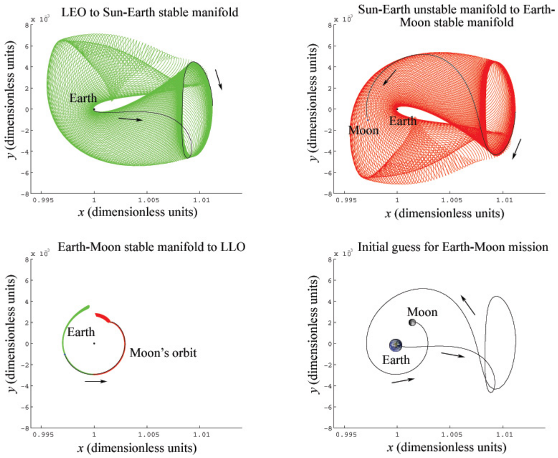
- Coordinate transformations
- Intersection interpolation
- Poincaré section
- Transfer region boundaries

Trajectory correction

- Four-body equations
- Optimization corrector

Results

Approximate trajectories in the four-body system



Conclusion

Discussion

- Summary of study
  - Periodic orbit extensions
  - Transfers between circular orbits and manifolds
  - Transfers between Sun-Earth and Earth-Moon manifolds

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- Provides a method to generate initial guesses for real low-energy Earth-Moon trajectories

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- More systematic than previous methods

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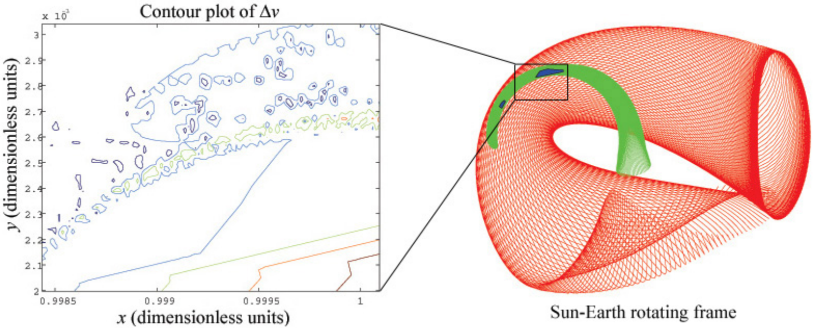
  - Transfers between circular orbits and manifolds
  - Transfers between Sun-Earth and Earth-Moon manifolds
- Provides a method to generate initial guesses for real low-energy Earth-Moon trajectories
- More systematic than previous methods
- Still need efficient correction routines

Conclusion

Future research: characterization of the Δv surface



Conclusion

Future research: transfer regions

- Manifold layers
  - Separate for smooth variation in Δv
  - Statistical clustering

Conclusion

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- Manifold layers
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- Geometry of low-energy transfer regions

Conclusion

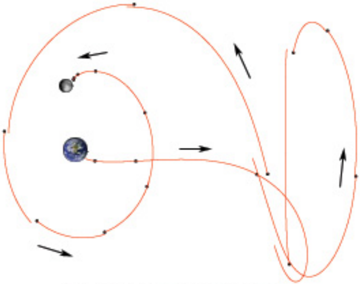
Future research: transfer regions

- Manifold layers
  - Separate for smooth variation in Δv
  - Statistical clustering
- Geometry of low-energy transfer regions
- Parametric study of the Δv surface

Conclusion

Future research: trajectory correction

- Form of merit function

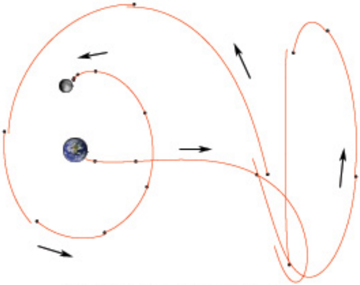


Sun-Earth rotating frame

Conclusion

Future research: trajectory correction

- Form of merit function
- Point sampling

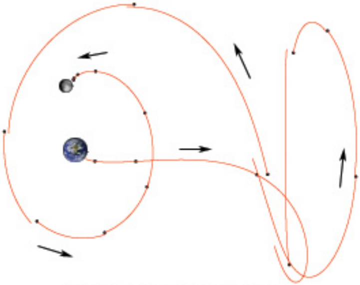


Sun-Earth rotating frame

Conclusion

Future research: trajectory correction

- Form of merit function
- Point sampling
- Computational efficiency

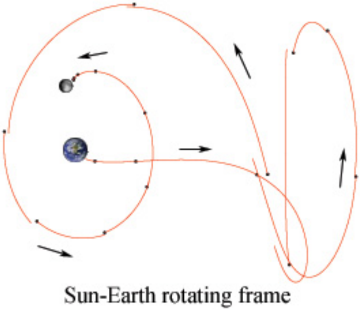


Sun-Earth rotating frame

Conclusion

Future research: trajectory correction

- Form of merit function
- Point sampling
- Computational efficiency
- Shooting method



Conclusion

Future research: model extensions

- More periodic orbits
  - L_1 Lagrange point
  - Retrograde orbits

Conclusion

Future research: model extensions

- More periodic orbits
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- Elliptical orbits

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Future research: model extensions

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- Elliptical orbits
- Orbital inclinations

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Future research: model extensions

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- Orbital inclinations
- Integration with JPL ephemeris

Conclusion

Future research: model extensions

- More periodic orbits
  - L_1 Lagrange point
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- Elliptical orbits
- Orbital inclinations
- Integration with JPL ephemeris
- 3D model extension

Acknowledgements

- Stefano Campagnola (faculty mentor)
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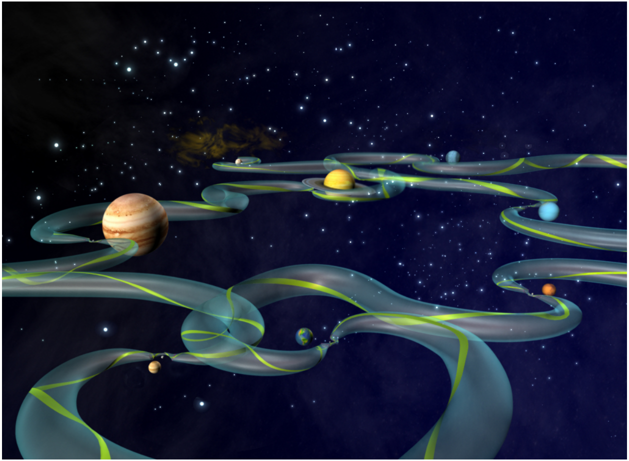
- Michael Raugh
- IPAM

Questions?



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Extra slides
Lagrange points

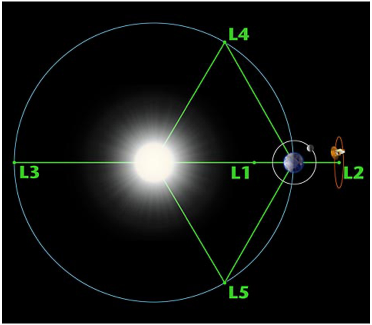

Image from NASA: `http://www.gsfc.nasa.gov/`.

Lagrange points

- Balance of gravitational and rotational forces
- Five equilibrium points
- Unstable (saddle): L_1, L_2, L_3
- Stable: L_4, L_5

Extra slides

Transfer region boundary polygons

- Spatial polygon: region of transfer in position space

Extra slides

Transfer region boundary polygons

- Spatial polygon: region of transfer in position space
- Total polygon: region of transfer in velocity space
  - Bounds all velocity coordinates corresponding to low Δv

Extra slides

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- Total polygon: region of transfer in velocity space
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- Intersection polygon: region in velocity space likely to contain zero-correction transfers

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- Total polygon: region of transfer in velocity space
  - Bounds all velocity coordinates corresponding to low Δv
- Intersection polygon: region in velocity space likely to contain zero-correction transfers
  - Polygons bounding velocity coordinates corresponding to low Δv for each manifold
  - Intersection of these polygons

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Transfer region boundary polygons

- Spatial polygon: region of transfer in position space
- Total polygon: region of transfer in velocity space
  - Bounds all velocity coordinates corresponding to low Δv
- Intersection polygon: region in velocity space likely to contain zero-correction transfers
  - Polygons bounding velocity coordinates corresponding to low Δv for each manifold
  - Intersection of these polygons
- All polygons are overestimations!

Extra slides

Density control by polygon construction

- Polygon construction
  - Triangulation, then polygon union
  - Convex polygon

Extra slides
Density control by polygon construction

- Polygon construction
  - Triangulation, then polygon union
  - Convex polygon
- Concave transfer region geometry

Extra slides

Density control by polygon construction

- Polygon construction
  - Triangulation, then polygon union
  - Convex polygon
- Concave transfer region geometry
- Concave polygon as union of convex polygons

Extra slides

Density control by polygon construction

- Polygon construction
  - Triangulation, then polygon union
  - Convex polygon
- Concave transfer region geometry
- Concave polygon as union of convex polygons
- Polygon construction by grid partition

Extra slides

Density control by polygon construction

- Polygon construction
  - Triangulation, then polygon union
  - Convex polygon
- Concave transfer region geometry
- Concave polygon as union of convex polygons
- Polygon construction by grid partition
  - No polygon constructed if less than two points in grid box!

Extra slides

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- Set tolerance for density of low-energy transfer points by appropriate gridding